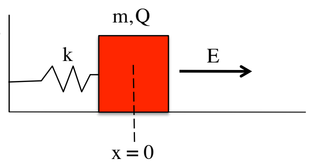
Problem 25.8

The system begins at the spring's equilibrium position.



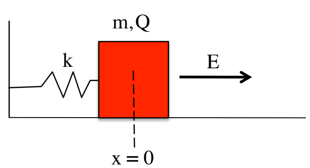
a.) The system is isolated in the sense that the only forces acting in the horizontal are the spring and the force generated by the electric field.

b.) There is spring potential energy and electrical potential energy in the system.

c.) There is a position where the electric force and the spring forces will exactly equal one another—the new equilibrium position for the spring/electric-field system. When the block gets to that position, momentum will carrying it through that position and out an equal distance on the other side of that position. In other words, if we can find that new equilibrium position, relative to the position we have defined as x = 0 (see sketch), doubling that will give us the momentum-equals-zero position. Doing so yields:

$$\sum F_x :$$
$$-kx_{new} + qE = m\cancel{a_x}^{\,0}$$
$$\Rightarrow \quad x_{new} = \frac{qE}{k}$$
$$\Rightarrow \quad x_{p=0} = 2x_{new} = 2\frac{qE}{k}$$

c.) As an alternative to the approach outlined above, we could do this using the idea of energy. Taking the electrical potential energy to be zero at the x = 0 position defined in the sketch, the voltage difference between that point and the p = 0 point (call this $x_{p=0}$) will be:

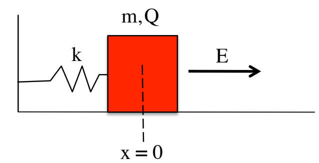


$$\Delta V = \frac{W_{\text{due to E}}}{q} = -\vec{E} \bullet d = -Ex_{p=0}$$
$$\Rightarrow \quad W_{\text{due to E}} = -qEx_{p=0}$$

We could use the conservation of energy at this point, but the work/energy theorem works just as well. Using that, we get:

$$W_{net} = \Delta KE$$
$$\Rightarrow \quad W_{\text{elect fld}} + W_{spring} = 0$$
$$\Rightarrow \quad -qEx_{p=0} + \frac{1}{2}k\left(x_{p=0}\right)^2 = 0$$
$$\Rightarrow \quad x_{p=0} = \frac{2qE}{k}$$

d.) The position where the new spring/electric field has zero net energy is called EQUILIBRIUM.



e.) We determined the new equilibrium position in the first attempt at *Part c*. That value was

$$x_{new} = \frac{qE}{k}$$

f.) Now the fun starts. If you will remember back to our discussions of vibratory motion, we observed that if we could take a Newton's Second Law equation and put it into the form $(\text{acceleration}) + (\text{constant})(\text{position}) = 0$, the motion was simple harmonic and the square root of the constant was the angular frequency of the motion. (For a straight spring system, the N.S.L. equation was put into the form

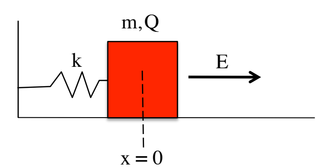
$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

and the angular frequency was found to be $\omega = \sqrt{k/m}$.

Starting with N.S.L. on the spring/electric field system when the block is at an arbitrary position

$$x_{new} + x$$

we can write:



$$\sum F_x :$$
$$-k\left(x_{new} + x\right) + qE = m\frac{d^2x}{dt^2}$$

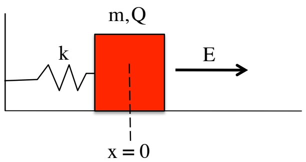
What's tricky about this is that we have already established that $kx_{new} = qE$ (see page one), so we can rewrite this as:

$$\sum F_x :$$
$$-k(x) = m\frac{d^2x}{dt^2}$$

Rearranging, we get:

$$-k(x) = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$



and voila, we have the characteristic form of a system that oscillates with simple harmonic motion, with the angular frequency being given by the relationship:

$$\omega = \left(\frac{k}{m}\right)^{1/2}.$$

g.) The period will be:

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega} = 2\pi\left(\frac{m}{k}\right)^{1/2}.$$

h.) All the electric field does is help define the new equilibrium position of the system. It does nothing beyond that. (This is a lot like a spring/mass system hanging in the vertical—gravity pulls the mass down to a new equilibrium position, but once there gravity does nothing to the oscillatory motion generated by the spring.)